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A note on the confinement of charged particles by a magnetic field

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With 2 figures in the text

A thermonuclear fusion reaction in an ionized gas may become self-sustained only if the gas is trapped very effectively inside the intended reaction volume. It has earlier been suggested (Lehnert 1958) that the motion of charged particles can be strongly confined in the magnetic field of a ring-shaped *solid* conductor, at least at low gas densities. This property is characteristic for any axisymmetric field which has a poloidal component (a component in the planes through the axis of symmetry) forming closed field lines inside the confinement volume.

The result is easily deduced from Poisson's bracket and the Hamiltonian H of a charged particle of mass m, charge q and velocity $\boldsymbol{v}$, which moves in the static magnetic field $\boldsymbol{B} = \mathit{curl}\ \boldsymbol{A}$ and the electrostatic field $\boldsymbol{E} = -\boldsymbol{\nabla}\phi$;

$$v_r^2 + v_z^2 = v_0^2 + \frac{2q}{m}(\phi_0 - \phi) - \frac{1}{r^2}\left[r_0 v_{\varphi_0} + \frac{q}{m}(r_0 A_{\varphi_0} - r A_\varphi)\right], \tag{1}$$

where (r, φ, z) are cylindrical coordinates. Index $(_0)$ indicates values at the starting point. In absence of a toroidal field the motion in the rz-planes takes place in an equivalent potential "trough" with a bottom line directed approximately along the field line $r A_\varphi = r_0 A_{\varphi_0}$, provided that the magnetic field is chosen strong enough. It should be observed that a complete confinement can be obtained with a poloidal field (B_r, B_φ), but not with a toroidal (B_φ). This result can also be deduced from Störmer's equations (Störmer 1955) or from the equation of motion (Lüst and Schlüter 1957).

Without violating the condition that the poloidal field lines should be closed inside the confinement volume part of the currents which generate the field may be situated outside the same volume, as shown by the spherical coil (d) in Fig. 1. In addition, a toroidal component may be added to form a screw field.

One of the essential difficulties with the present arrangement is caused by the leads for the supply current to the loop (a) which also provide its mechanical mounting. In Fig. 1 the leads have been placed in the plane of the current loop and Fig. 2 shows their cross section. First consider the plane case of infinitely long leads in a homogeneous field $\boldsymbol{B}_0$. The ratio $F = b_1 / B_0$ between the self-field b_1 at the surface of a lead and B_0 is chosen equal to $2d_1 / r_1$, where $2d_1$ is the distance between the centra of the leads and r_1 the radius of a lead. Then, the resulting force on each lead in the horizontal plane vanishes and the leads are encircled by a magnetic field which at the

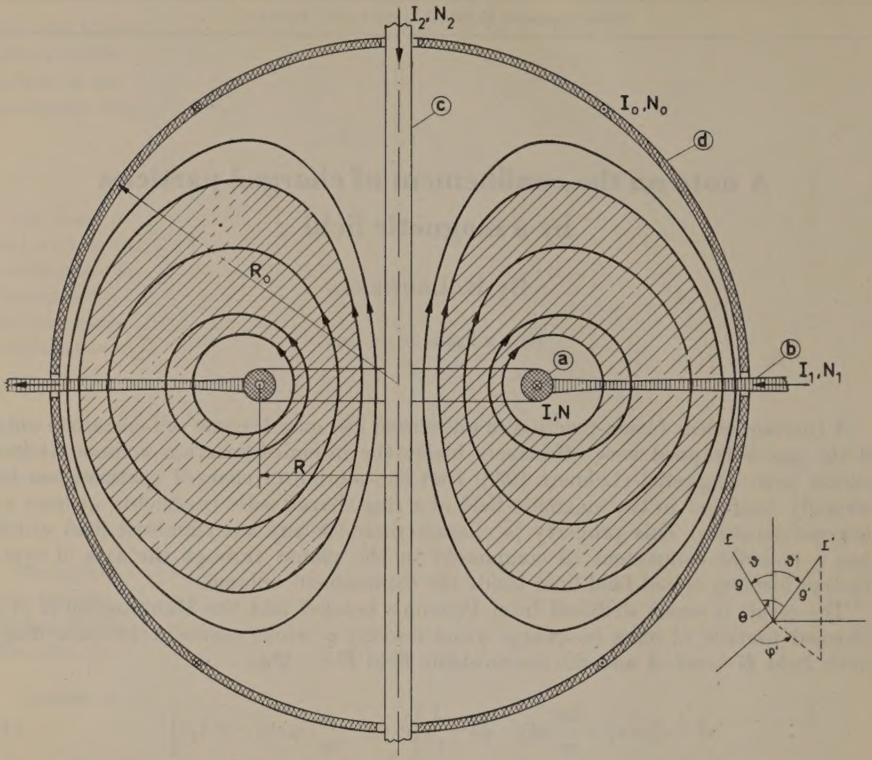

Fig. 1. The magnetic field generated by a circular coil (a) of radius R and a spherical coil (d) of radius R_0, where the field of the latter cancels that of the former outside the sphere $\varrho = R_0$. Current is supplied to the coil (a) by the leads (b). A toroidal field may be generated by the conductor (c) to form a screw field.

same time screens them from particle bombardment. A particle which starts at least outside the boundary C_0 will not reach the lead surface if

$$(1 + u_0/v_0)\,(a_1/r_1) < \log\,(F - 1) + \frac{1}{F} - \frac{1}{2}, \tag{2}$$

where $a_1 = mv_0/|q|\,b_1$, $u_0^2 = v_0^2 + 2\,q\,(\phi_0 - \phi)/m$ and reflection of particles is considered under the most unfavourable situation where $q\,(\phi_0 - \phi) > 0$.

The lines ($\xi = 0$, $\eta = \pm\sqrt{3}\,F/2$) where the magnetic field vanishes might be assumed to cause a particle leakage in the direction out of Fig. 2. However, it can be shown that a particle which starts at some distance from a zero line cannot reach the line with a vanishing velocity in the plane of the figure. The only way for the particle to get lost in the direction of the leads is by a collision in the immediate neighbourhood of a zero point by which the particle is scattered into a very narrow solid angle centered around the zero line. Consequently, this particle leakage will be

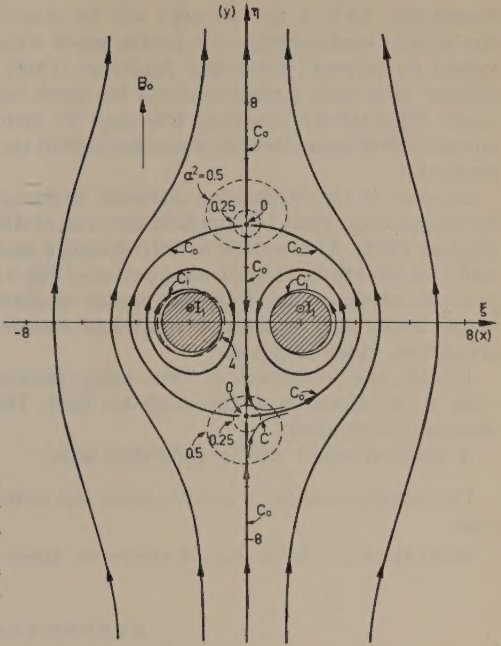

Fig. 2. The magnetic field in the vicinity of the leads. $\boldsymbol{B}_0$ is the field generated by other sources than the leads. C_0 forms the boundary of the field lines which encircle the leads. Dotted lines give regions inside of which the total magnetic field strength is less than $\alpha^2 B_0$. The figure demonstrates the case $F = 4$.

extremely small for the relatively long mean free paths which are of interest under thermonuclear conditions.

That the leads will reflect particles strongly also in the more general case of Fig. 1 is made plausible by the fact that the lines $B =$ const in the immediate neighbourhood of the leads very nearly coincide both with the field lines and with the lead surfaces (see Fig. 2). From Alfvén's perturbation theory (Alfvén 1950) this implies that the motion along the magnetic field as well as the drift motion caused by its inhomogeneity is directed almost parallell with the lead surface.

The field inhomogeneities introduced by the currents through the leads produce additional drift motions. However, for particles running around the configuration repeatedly these additional motions cancel.

Practically any heating mechanism suggested earlier can be used in combination with the magnetic "bottle" discussed here. One possible way is to inject ions from an ion source into a screw field which varies slowly in time. This prevents ions from hitting the back side of the source, both after the first turn around the winding of the circular coil and after the first passage around the axis of symmetry. It is also possible to heat the plasma by means of an applied poloidal electric field which sets the plasma into rotation, similar to what has been done in the "Ixion" (Boyer 1957). Finally, any oscillating electromagnetic field will heat the plasma, provided that the confinement is perfect (Spitzer and Witten 1953, Berger *et al.* 1958).

The field configuration suggested here is likely to provide a strong particle confinement at low gas densities. At high densities, however, its stability has also to be considered. Only a few remarks will be made here. A straight correspondence to the present configuration is a hollow pinch which encloses a straight conductor traversed by current. Baker and Anderson (1956) have performed experiments which indicate that such a pinch is stable for much longer periods than a simple pinch filament. Figuratively speaking, this may be undersood by the fact that part of the current which generates the magnetic field in the plasma is "anchored" in the central conductor.

In some of the Stellarator machines exchange instabilities have been suppressed by introducing shear, i.e., the field lines run at different pitch angles in different layers (Spitzer 1956). This is done by introducing a small poloidal perturbation on an essentially toroidal field. In the configuration of Fig. 1 a shear can be produced which is more pronounced than that of the Stellarator machines. The arrangement, with inclusion of the leads, also produces a rotational transform which short-circuits any charge separation which may arise.

Finally, the production of " run-away electrons" requires an electric field component in the direction of the magnetic field. This component can be avoided in the present arrangement.

A detailed report will be published later.

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